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# MEASUREMENT OF MOIST AIR VELOCITY BY MEANS OF HOT-WIRE ANEMOMETERS


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The effect of air humidity on the accuracy of measurements by means of constant-temperature hot-wire anemometers is investigated, and a method for reducing the resulting error is described.


The hot-wire anemometer is presently one of the most widely used instruments for turbulent flow measurements. There is a large number of publications on the most diverse aspects of hot-wire anemometer use [1]. Papers on interpretation of hot-wire anemometer measurement results for a wide range of medium parameters, such as the pressure, the density, the temperature, etc. [2], are published regularly. Nevertheless, the effect of humidity on the accuracy in measuring the velocity characteristics of gaseous media by means of hot-wire anemometers has not received enough attention in the domestic literature.

We shall consider here a "constant-temperature (resistance)" hot-wire anemometer (CTHWA) [3, 4], which has lately been used widely in practice, especially for measurements in air [5, 6].

Neglecting the effect of the filament's thermal radiation, which is indeed proper for air at the usual operating temperatures in the 250-300°C range [1, 3], we write the semi-empirical Kramers expression for heat exchange under steady-state conditions [3] for the filament:

$$U^2 = \frac{R_w (R_w - R_g)}{bR_0} l\pi [K_f^{0.8} (c_p\mu_f)^{0.2} + 0.57K_f^{0.67} (c_p\mu_f)^{0.33}(\rho_f dv/\mu_f)^{0.5}], \tag{1}$$

where b is the thermal coefficient of the filament resistivity if we assume that the temperature dependence of the resistance $R_w$ is linear, i.e., $R_w \approx R_0[1 + b(\theta_w - \theta_0)]$, where $R_0$ is the filament's resistance at $\theta_0$ = 273°K.

It is evident that the values of $K_f$, $\mu_f$, $c_p$, and $\rho_f$ for air depend on the amount of water vapor in the air. Let us determine the form of these relationships and write, in the final analysis, expression (1) as a function of the humidity.

The expressions for $K_f$ and $\mu_f$ are known from the kinetic theory of gases [7] (since our considerations pertain to virtually normal conditions, we shall henceforth consider air and the water vapor in it as an ideal gas):


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$$K_f = \frac{1}{3} \bar{v}_M l_{mfp} \rho_f c_V, \tag{2}$$

$$\mu_f = \frac{1}{3} \bar{v}_M \rho_f l_{mfp}, \tag{3}$$

$$\bar{v}_M = \sqrt{\frac{8R\theta_f}{\pi\mu}}, \tag{4}$$

where $l_{mfp}$, $\bar{v}_M$, $c_V$, and $\mu$ are the mean free path, the mean molecular velocity, the specific heat for constant volume, and the molecular weight, respectively. We have $\rho_f l_{mfp} \approx$ const for this complex, since $l_{mfp} \sim 1/\rho_f$ [8] under ordinary conditions. We denote by q the percentage of water vapor per unit mass of moist air:

$$q = \frac{\rho_V}{\rho_V + \rho_a} = \frac{\mu_V}{\mu_a} \left[ \frac{e}{p - (1 - \mu_V/\mu_a) e} \right], \tag{5}$$

where p, $\rho_V$, $\rho_a$, $\mu_V$, and $\mu_a$ are the pressure of the mixture, the vapor density, the air density, the molecular weight of vapor, and the molecular weight of air ($\mu_V = 18.016$; $\mu_a = 28.966$). Expression (5) was derived under the assumption that

$$e = \frac{R}{\mu_V} \rho_V \theta_f, \quad p - e = \frac{R}{\mu_a} \rho_a \theta_f, \tag{6}$$

where R is the universal gas constant, $R = 8.31 \cdot 10^3$ kJ/kmole·deg, and $\theta_f$ is given in degrees Kelvin. The mixture density $\rho_f$ is then given by

$$\rho_f = \frac{\mu_a}{R\theta_f} [p - ae], \tag{7}$$

where $a = 1 - \mu_V/\mu_a = 0.38$.

As for $c_V$ and $\mu$ for a mixture of gases, such as air, they are treated as certain relative characteristics and are expressed in terms of the specific heat values $c_{Vi}$ and the molecular wieghts $\mu_i$ of real gases, which form parts of the mixture additively (with regard to their weight, i.e., percentage, content $q_i$ in the mixture):

$$c_V = \sum_i q_i c_{Vi} \text{ and } \mu = \sum_i q_i \mu_i, \tag{8}$$

i.e.,

$$c_V = c_{V_V} q + c_{V_a}(1 - q) \text{ and } \mu = \mu_V q + \mu_a (1 - q). \tag{9}$$

It is considered that only the water vapor concentration in air is variable (the absolute content of carbon dioxide in air is small in comparison with that of water vapor and, therefore, the contribution of its fluctuations to (9) can be neglected [9]). The quantity q in (9) has the sense defined above. It should also be noted that, with an error negligible for our purposes, the $c_{V_V}$ value for water vapor is approximately equal to $2c_{V_a}$ (if we neglect the weight percentages of other triatomic gases in air), so that after simple, although cumbersome, calculations, we obtain

$$c_V \approx \frac{c_{V_V}(1 + 0.24e/p)}{2(1 - ae/p)}, \quad \mu \approx \frac{\mu_V (1 - 0.62e/p)}{0.62 (1 - ae/p)}. \tag{10}$$

These expressions are different from the results obtained in [10], where the assumption of a linear relationship between $\mu$ and $c_V$ of the following form has been put forward: $\mu \sim (1 + h_k e/p)$, where $h_k$ varies from $-0.13$ to 1.17.

Substituting (7) and (10) in (1) and considering that, according to Meyer's equation [7, 8], $c_{pa} = 1.4c_{Va}$ and $\Delta\theta = \theta_w - \theta_g = (R_w - R_g)/bR_0$ for an ideal gas, such as dry air, we find that the output signal E for CTHWA depends on e in the following manner:

$$E \sim U \sim \{A(1 + 0.58e/p) + B(1 + 0.52e/p)v^{0.5}\}^{0.5}, \qquad (11)$$

where

$$A \approx 0.197 R_w \sqrt{R}\, c_{V_a} (l_{mfp}\rho_f)^2 l \Delta\theta \sqrt{\theta_f};$$

$$B \approx 0.186 R_w l (l_{mfp}\rho_f)^2 \Delta\theta c_{V_a} (d\mu_a p)^{0,5} \mu_v^{-0,25}.$$

It is evident from (11) that E increases with e (and $\theta$), since $e = re_s(t)$, where r is the relative humidity, and $e_s(t)$ is the saturating vapor pressure at a certain temperature t. The relationship $e_s(t)$ is described by the expression $e_s(t) = e_{s0}10^{7.45t(235+t)}$, where t is the the temperature (°C), and $e_{s0} = 6.11$, mb $= e_s(t)|_{t=0\,°C}$ [9]. This expression entails an error of less than 0.1 mb in the temperature range from $-50$ to $+50$°C, which is of practical importance.

Assume that $t = +30$°C and that the hot-wire anemometer was calibrated at $t = +15$°C and $r = 30\%$. Then, if r increases, for instance, to 95%, $\Delta e$ amounts to $e|_{r=95\%,\ t=+30°C} - e|_{r=30\%,\ t=+15°C} \geqslant 35$ mb, and, for p, v = const, it is evident from (11) that the variation $\Delta E = f(e)$ amounts to approximately 4% for ordinary atomospheric pressure values p = 900-1000 mb (which corresponds to the same errors with respect to the velocity for which the slope of the CTHWA calibration curve is $s \approx 1$).

These evaluations were checked experimentally in the following manner. The sensing element (filament) of the CTHWA was placed in the flow inside a wind tunnel, which was located within an air volume of roughly 1 $cm^3$ in the test compartment of a KTK-800 climatic thermal chamber [11]. The air temperature and humidity in the climatic chamber and the flow velocity in the tunnel were varied; the signal E was recorded by means of a V2-23 integrating digital voltmeter. The temperature was checked by means of a special resistance thermometer (RT) [6], the sensing element of which was located at a distance of $\sim 10^{-2}$ m from the CTHWA sensing element. The RT signal was also used for compensating thermal variations in the ohmic resistance of the CTHWA sensing element according to the method described in [6]. The flow velocity in the tunnel was changed by varying through remote manipulation the operating voltage arriving at the motor of the tunnel fan from a ferroresonance regulator. The velocity was monitored by means of a Pitot tube with an inclined alcohol pressure gauge. The tube was placed at the location of the CTHWA and RT sensing elements, which consisted of Wollaston laminar filaments with $d = 16 \cdot 10^{-6}$ m. They were welded to a stainless-steel $10^{-2} \times 10^{-2}$ frame-shaped support with $d = 0.8 \cdot 10^{-3}$ m and fastened in a ceramic tube with $d = 0.5 \cdot 10^{-2}$ m and a length of approximately $10^{-1}$ m. In this $R_g|_{t=+20\pm0,5\,°C} \approx 5$ $\Omega$ for CTHWA and $R_0|_{t=0\pm0,02\,°C} \approx 50$ $\Omega$ for RT. The overheat was $(R_w - R_g)/R_g = 1$. The electronic components of CTHWA and RT were thermostatically controlled with an accuracy not worse than +55° ± 0.3°C within a single casing with a volume of about $10^{-3}$ $m^3$, which permitted us to neglect the temperature drift of the electronic devices and avoid inductions.

The velocity fluctuations in the tunnel caused by the fans of the climatic thermal chamber or, more accurately, the corresponding fluctuations of E were smoothed out before measurements by means of a passive RC-network with the time constant $\tau = 3.2$ sec and a compensating device based on a K140UD7 operating amplifier [12].

The processing of experimental data was performed by building up collections of $E_i$ readings with the sampling period T = 6 sec $\approx 2\tau_{RC}$, which ensured uncorrelated samples (the process is considered to be steady-state and ergodic, i.e., $T = 2\tau_{RC} \gg 1/4\Delta F$ [13]; in our case, $\Delta F \approx 1.5$ kHz is the band of frequencies measured without distortion by the CTHWA variant used) for any fixed value of v, e, and $\theta$; the processing also included evaluation of the mathematical expectation $(\sum_{i=1}^{n} E_i)/n$, where n corresponded approximately to a 5-min interval.

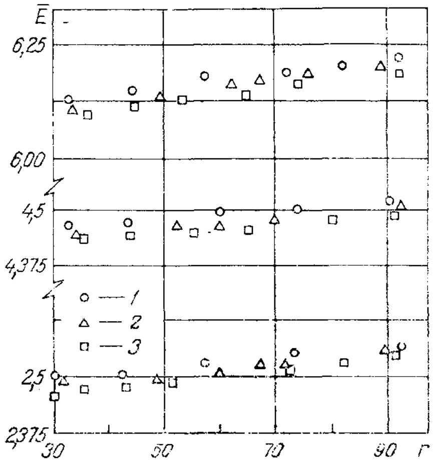


Fig. 1. Behavior of the output signal of the hot-wire anemometer for different velocity, temperature, and humidity values ($\bar{E}$ is the mean value of the output signal (V); r is the relative humidity (%); $v_1$ = 0.64; $v_2$ = 2.75; $v_3$ = 6.35 m/sec). 1) +35; 2) +27; 3) +15°C.

Figure 1 shows the results of measuring the CTHWA output voltage as a function of the humidity for different flow velocities in the tunnel. It is evident from the diagrams that the mean values $\bar{E}|_{v=\text{const}}$ are described almost exactly by a function in the form ke, where k is a positive proportionality factor, while e varied from 5.1 mb at +15 ± 0.1°C to 40.5 mb at +35 ± 0.1°C. Using the calibration curve plotted for t = 27 ± 0.1°C and r = 70%, we found for every velocity value the error of velocity determination $\Delta v = \Delta E(e)/sv \cdot 100\%$, where the values of s were respectively equal to 25.5 (E = 2.55 V); 8.70 (E = 4.45 V); 4.64 mV·sec/cm (E = 6.13 V). The Δv values for these velocities: 7.3; 5.5; 5.6% are in good agreement with the above evaluations obtained by means of expression (11). The increase in the error at low velocities can be explained by the rising inaccuracy of CTHWA calibration at velocities lower than 1 m/sec. Summarizing, we note that the obtained mean "error sensitivity" with respect to velocity of the CTHWA amounts to ~0.16%/mb.

These estimates generally agree with the results from [14], where the following relationship was obtained: A change in humidity by 15 mb entails an increase in the error of v determination by not less than 1%.

It is understood that, in most practical cases, the error in measuring Δv as a function of Δe can be reduced to a minimum by the optimum choice of moisture in the gas during calibration (it is desirable that the moisture be equal to the mean expected value $\bar{e} = e_{\min} + \frac{\Delta \bar{e}_{\max}}{2}$) These experiments suggest one more way of reducing Δv(Δe), and that is to operate on a segment with a sufficiently steep slope s (for an acceptable accuracy, even under conditions where the humidity of the ambient is not known *a priori*, one should strive to ensure $s \gg 1$ in the required velocity range).

## NOTATION

v, flow velocity; r, relative humidity; U, voltage drop at the filament; $R_w$, $R_g$, and $R_o$, resistances of the "overheated" filament, filament at the gas temperature, and filament at 0°C, respectively; $\theta_w$ and $\theta_g$, temperatures of the filament and the gas, respectively; $c_p$ and $c_v$, specific heat values for constant pressure and constant volume, respectively; μ, dynamic viscosity; ρ, density; $\mu_v$, $\mu_a$, and μ, molecular weights of vapor, dry air, and moist air, respectively; $K_f$, thermal conductivity coefficient for air; b, thermal coefficient of filament resistance; $\bar{v}_M$, mean molecular velocity; R, universal gas constant; p, pressure; e, absolute vapor pressure; t, temperature (°C); T, period; $\tau_{RC}$, time constant of the RC-network; ΔF, frequency band; s, slope of the hot-wire anemometer's calibration curve; E, output signal of the hot-wire anemometer. The subscript f signifies that a quantity pertain to the film temperature $\theta_f = (\theta_w + \theta_g)/2$.

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